

## Analysing uncertainty contributions in dimensional measurements of large-size objects by ultrasound sensors

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According to the ever-increasing interest in metrological systems for dimensional measurements of large-size objects in a wide range of industrial sectors, several solutions based on different technologies, working principles, architectures and functionalities have been recently designed. Among these, a distributed flexible system based on a network of low-cost ultrasound (US) sensors – the Mobile Spatial coordinate Measuring System (MScMS) – has been developed. This article presents a possible approach to assess the system uncertainty referring to the measured point coordinates in the 3D space, focusing on the sources of measurement uncertainty and the related propagation law.

**Keywords:** uncertainty; coordinate measurement system; large-scale dimensional metrology

### 1. Introduction

Large-scale dimensional metrology (LSDM) is that branch of metrology that deals with the measurement of objects whose dimensions range from few metres up to tens of metres (Puttock and Lorenz 1978). Because of the size of the involved volumes, large-scale measurements can hardly be performed in a well-controlled environment and often the metrologists are forced to modify or adapt the equipment and the approaches to suit the circumstances and to achieve the desired accuracy (Franceschini et al. 2011). Given its potential in industrial applications, the field of LSDM has recently arisen a burgeoning interest. On the one hand, technological progress has enabled the development of new metrology systems capable of dealing with measurements of large-size objects; on the other hand, such systems have been increasingly applied in industrial environments in order to satisfy the pressing constraints imposed by the latest specifications (Peggs et al. 2009).

Because of the variety of the technologies and the dimensions of the working volumes, now one of the outstanding problems in the field of LSDM is the estimation of the measurement uncertainty. For this reason, the standard references in this regard are very few.

After introducing an LSDM system developed at the quality and industrial metrology laboratories of Politecnico di Torino, this article presents a model for its uncertainty assessment. The prototype system is described in Section 3. In Section 4, the major uncertainty contributions are analysed and the uncertainty model is presented. Some verification and application tests are shown in

Sections 5 and 6. The concluding section highlights the main implications, limitations and original contributions of this article.

### 2. State of the art

Several works have periodically tried to take stock of the developments in the field of LSDM, both in terms of instrumentation and regulatory standards that are accompanying the evolution of the technology (Peggs et al. 2009).

From the technological point of view, over the past 20 years, significant developments have led to the development of new metrology systems, building on already existing technologies or applying completely new working principles (Estler et al. 2002; Peggs et al. 2009). Luhmann et al. in their work provided a qualitative assessment of major systems according to both working volume and accuracy (Luhmann et al. 2006; Luhmann 2010). The results of this survey are summarised in Figure 1.

Due to this variety, the choice of the ‘best’ system for all measurement tasks is hardly possible (Cai et al. 2009). Cuypers et al. identify many factors involved in the selection of a system instead of another one: a part from the investment cost, task requirements (accuracy, reliability of results, number of points, etc.), environmental constraints (temperatures, vibrations, humidity, etc.) and measured object restrictions (dimensions, material, surface quality, etc.) have to be considered (Cuypers et al. 2009). Muelaner et al. also deal with the problem of the choice of the more appropriate measuring system, proposing, in

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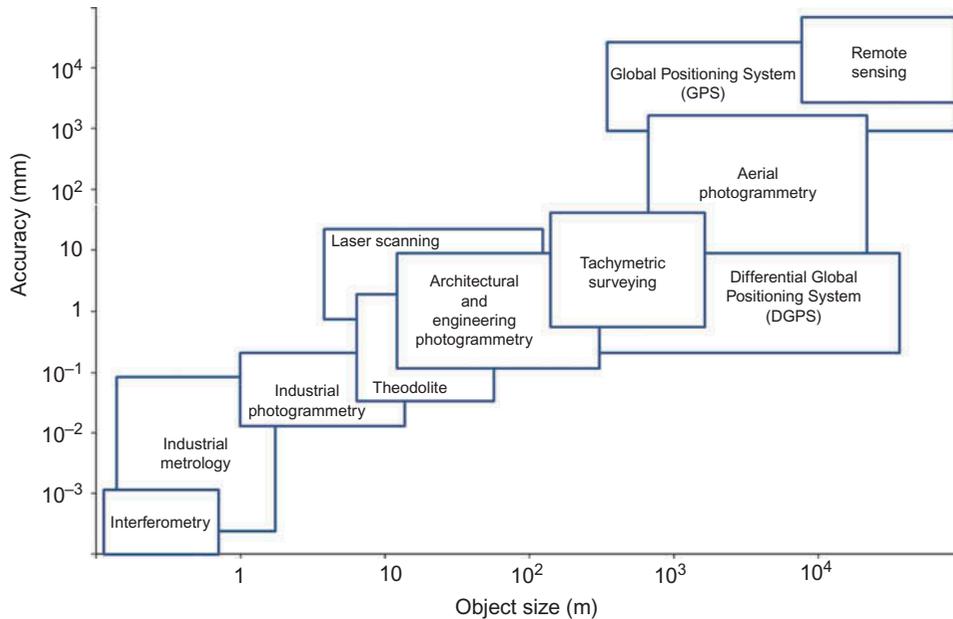


Figure 1. Qualitative relationship between working volume and accuracy of different systems/technologies Luhmann et al. (2006).

their studies, a prototype system selection as the basis for development of a more sophisticated measurement planning tool (Muelaner, Cai, and Maropoulos 2010).

From the point of view of the development of a common set of standards for the evaluation of systems' uncertainty, research has lagged behind. Regulations in this field have tried to run after the rapid evolution of the systems; however, their updating has been complicated by the discussed diversity of the technologies and working principles. Additionally, the strides to merge existing solutions in order to increase ease of use and metrological performance of the systems brought to a further complication (Weckenmann et al. 2009; Franceschini et al. 2011; Maropoulos et al. 2011).

So far, while well-established standards exist for coordinate measuring machines (CMMs), the state of the art and the effective reference for researchers and manufacturers in LSDM are represented by the general metrology standards (JCGM100:2008 2008; JCGM200:2008 2008) and a few specific standards for existing systems or optical CMM (VDI/VDE2634 2002; Peggs et al. 2009; ISO15530-3:2011 2011).

Nevertheless, the knowledge of the performance characteristics of a system is something which cannot be ignored in manufacturing. In fact, each measurement is meaningless without an indication of uncertainty of the system (Lira 2002; Cox and Harris 2006; Franceschini et al. 2009a). On the other hand, trusting the performance claims of the manufacturers of the measuring systems can be dangerous, especially for those systems for which the working volume is highly anisotropic (Maropoulos et al.

2008; Galetto, Mastrogiacomo, and Pralio 2010; Wang et al. 2011).

In a context of partial absence of specific reference standards, the study presented in this article is an attempt to overcome this lack. This article presents a model able to provide an estimation of the uncertainty related to a 3D point measurement performed by contact systems based on multilateration. The model was developed for a specific system: the Mobile Spatial coordinate Measuring System (MScMS) (Franceschini et al. 2009b). It is a prototype system based on a network of US sensors distributed in a working environment. The prototype was expressly developed with the aim of fulfilling some important requirements of portability, flexibility and ease of use many other systems in the field of LSDM are not able to guarantee (Franceschini et al. 2009c). Being composed of light and self-powered network devices, the system is able to be easily moved, installed and used in different operational environments. In addition, the redundancy of information deriving from its network architecture makes the system extremely reliable even in harsh operating conditions. Its distributed nature enhances the system with a further interesting feature: the ability to customise the working volume depending on the applications, the object to be measured and the obstacles in the environment (Peña, Krommenacker, and Charpentier 2011; Huang et al. 2012).

In spite of these appealing features, compared to other systems, MScMS still offers lower metrological performance. Yet its cost and versatility may justify its usage in those areas where accuracy requirements are less severe. To date, however, it is difficult to determine in

advance for which tasks the system can be considered suitable. This is because the system presents metrological performance highly inhomogeneous in the working volume which significantly complicates the assessment of its accuracy.

The model presented in this article tries to address this issue. Further, the model could be helpful for the optimisation of the measurement task, aiding, for instance, the user in the definition of the position of the object to be measured or in the selection of the measurement points. Although being a system-specific model, this research represents a valuable approach that can be extended to similar systems for which the working volume can be highly anisotropic.

### 3. About MScMS

MScMS is a prototype, made up of three main components (see Figure 2) (Franceschini et al. 2009a):

- A constellation (network) of wireless devices, opportunely arranged around the working area (Galetto and Pralio 2010)
- A measuring probe, provided with other two wireless devices, communicating via US transceivers with the constellation devices in order to obtain their reciprocal distances
- A computing and controlling system (PC), receiving and processing data sent by the mobile probe in order to compute the coordinates of the points touched by the probe and evaluate object's geometrical features

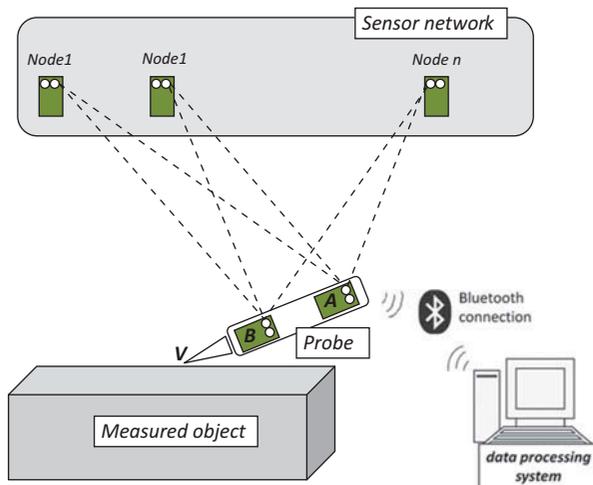


Figure 2. Schematic representation of the MScMS architecture. The dashed lines represent measured distances between sensor and probe nodes (indicated as A and B) equipping the hand-held probe. A Bluetooth connection is established between the probe and the processing system.

Crickets, developed by the Massachusetts Institute of Technology and Crossbow Technology Inc., have been adopted as wireless devices both for the constellation and for the probe. They utilise one radiofrequency (RF) and two US transceivers in order to communicate and evaluate mutual distances. Mutual distances are estimated by a technique known as TDoA (Time Difference of Arrival) (Gustafsson and Gunnarsson 2003). The RF communication makes each Cricket rapidly know the distances among other devices.

More specifically, the measuring probe is a mobile system hosting two Crickets, a tip to touch the surface points of the measured objects and a trigger to activate data acquisition (Franceschini et al. 2009b). Constellation devices operate as reference points (beacons) for the mobile probe. Spatial location and calibration of the constellation devices are made up by a specific procedure using a multilateration technique (Lee and Ferreira 2002; Franceschini et al. 2009c; Mastrogiacomo and Maisano 2010). A Bluetooth transmitter connected to one of the two probe's Crickets sends the measured distances to the PC, equipped with an ad hoc software to elaborate them.

Given the geometrical characteristics of the mobile probe, the tip coordinates can be univocally determined by means of the spatial coordinates of the two probe Crickets.

In this way, the system makes it possible to calculate the position – in terms of spatial coordinates – of the object points ‘touched’ by the probe. Then different types of elaborations can be performed: determination of distances, geometrical tolerances, geometrical curves or object surfaces.

### 4. Uncertainty evaluation of MScMS measurements

Various approaches for the assessment of the uncertainty related to the measurement of a 3D point using multilateration techniques are suggested in scientific literature (Sommer and Siebert 2006; Peggs et al. 2009; Weckenmann et al. 2009). Basically two alternative approaches to uncertainty assessment exist:

- Multivariate Law of Propagation of Uncertainty (JCGM100:2008 2008)
- Monte Carlo Sampling Technique (Peggs et al. 2009)

The Multivariate Law of Propagation of Uncertainty (MLPU) applies to a measurand  $\mathbf{Y} = [Y_1 \ \dots \ Y_M]^T$ , which is determined as a function of  $N$  other quantities  $\mathbf{X} = [X_1 \ \dots \ X_N]^T$ :

$$\mathbf{Y} = \mathbf{f}(\mathbf{X}) \quad (1)$$

Expanding  $f(\mathbf{x})$  of Equation (1) into a first-order Taylor series around the average values  $\bar{\mathbf{x}}$  of the estimates vector  $\mathbf{x}$ , MLPU can be used to achieve an estimate of covariance matrix  $\Sigma_Y \in \mathbb{R}^{M,M}$  associated with  $Y$  (Kacker, Sommer, and Siebert 2007; JCGM100:2008 2008; Weckenmann et al. 2009):

$$\hat{\Sigma}_Y = \mathbf{J}\Sigma_X\mathbf{J}^T \quad (2)$$

where  $\mathbf{J} \in \mathbb{R}^{M,N}$  is the Jacobian of  $f(x)$  and  $\Sigma_X \in \mathbb{R}^{N,N}$  is the covariance matrix of  $X$ .

Notice that the application of MLPU does not require any assumptions about the distribution of these contributions, but just some knowledge about their respective variance(s) and covariance(s) of their distribution.

The approach presented in this article is based on MLPU because, even if through a linearisation, this method allows to tackle the problem from a theoretical point of view, highlighting all the various contributions of uncertainty.

Referring to MScMS, the main contributions to the overall uncertainty can be identified in Franceschini et al. (2011):

- (1) Uncertainty of the probe reference points  $A \equiv (x_A, x_A, x_A)$  and  $B \equiv (x_B, x_B, x_B)$  (see Figure 2), which can be ascribed to the localisation algorithm. In detail, this contribution to uncertainty can be charged to other factors:

- (a) Uncertainty of measured distances ( $d_i$ , with  $i = 1, \dots, n$ ) from each of the two probe Crickets (A and B) to each of the  $n$  network devices
  - (b) Uncertainty of the coordinates of each Cricket of the constellation
  - (c) Synchronisation error among Crickets, which is negligible in static conditions
  - (d) Uncertainty of the parameters defining bias corrections to Cricket measurements (Maisano and Mastrogiacomo 2010)
- (2) Uncertainty related to the calibration of probe geometric parameters, that is, the distance between the two Crickets of the probe ( $d(A-B)$ ) and the distance between the two devices and the probe tip ( $d(A-V)$ ,  $d(B-V)$ )

Figure 3 schematically shows how the uncertainty contributions combine with each other. To make this discussion simple and general at the same time, in this study, it was decided to limit the uncertainty composition chain to these contributions.

The purpose of this article is to describe a model able to quantify the impact of the various contributions on the instrument measurement uncertainty. The model must express the uncertainty on the measured point ( $V$ ) as a function of the uncertainty related to calibration parameters ( $\xi$ ), measured distances ( $d$ ) and the probe geometric parameters:

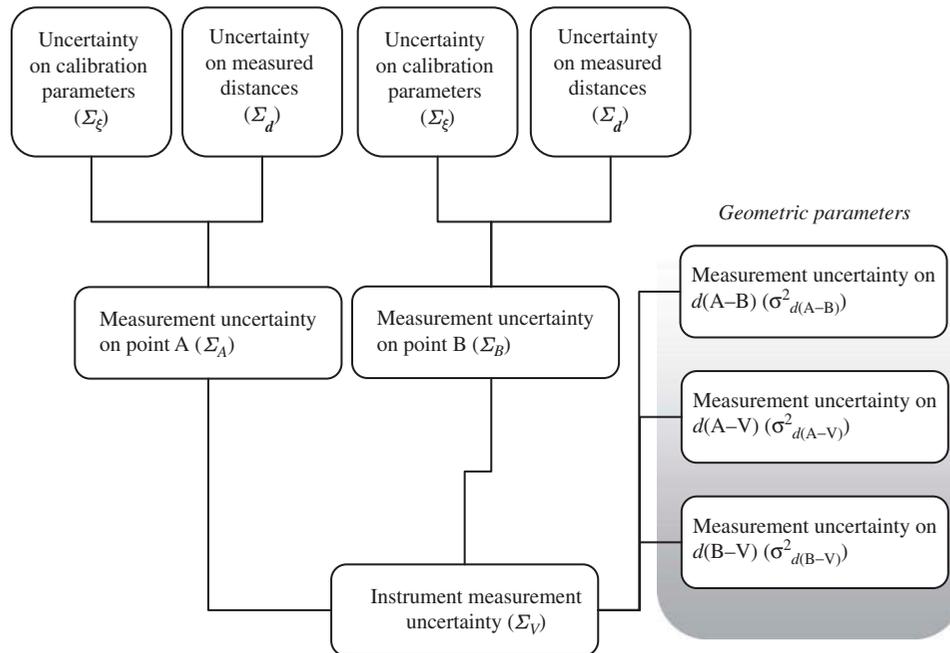


Figure 3. Uncertainty composition scheme.

$$\Sigma_V = f\left(\Sigma_\xi, \Sigma_d, \sigma_{d(A-B)}^2, \sigma_{d(B-V)}^2, \sigma_{d(A-V)}^2\right) \quad (3)$$

where  $\Sigma_V$ ,  $\Sigma_\xi$  and  $\Sigma_d$  are respectively the covariance matrix of  $V$ ,  $\xi$  and  $d$ , while  $\sigma_{d(A-B)}^2$ ,  $\sigma_{d(B-V)}^2$  and  $\sigma_{d(A-V)}^2$  are the variances related to the measurement of  $d(A-B)$ ,  $d(B-V)$  and  $d(A-V)$ , respectively.

#### 4.1. Uncertainty of measured distances

The probability density function of measured distances is very complex. In fact, the calculation of the distances through ultrasonic sensors was proved to depend on many factors, including

- Temperature and humidity: Environmental conditions significantly affect the velocity of propagation of US signals.
- Relative position of the US sensors: US transducers are not punctiform; also emitted US signal is generally directional. For this reason, the relative position of the transducers has an effect on distance estimation.
- State of charge of the measurement devices.

A comprehensive analysis of these factors is given by Franceschini et al. (2011). In addition to this analysis, Maisano and Mastrogiacomo (2010) proposed an iterative regressive model to compensate for the effect of the main factors. Despite the corrective models introduced, influences due to factors of difficult control (such as thermal gradients, layers of different humidity, velocity of air flow, and composition of the air) still remain as noise contribution to the measurements. Considering that the associated uncertainty is the result of the composition of several factors, distance measurement is assumed to be normally distributed with mean equal to the true value and standard deviation estimated whether through *a priori* or *a posteriori* considerations.

The *a priori* estimation can be performed through preliminary characterisation tests. A rough estimation of standard uncertainty (i.e., standard deviation) in normal environmental conditions was found to be lower than 5 mm in the entire range of measurement (Balakrishnan et al. 2003). Neglecting, in a first approximation, the correlation between the measured distances from a point  $x_P \equiv (x_P, y_P, z_P)$  to the  $n$  network devices, the covariance matrix  $\Sigma_d \in \mathbb{R}^{n,n}$  can be stated as the product between a scalar  $\sigma_0^2$  (e.g., the *a priori* estimation of the variance) and the identity matrix  $I$ :

$$\hat{\Sigma}_d = \sigma_0^2 I \quad (4)$$

The *a posteriori* estimation of the covariance matrix  $\Sigma_d$  is certainly a viable alternative to the *a priori* approach. In

quasi-static conditions, each measurement is generally the result of the average of a number of acquisitions. These replications allow to obtain an estimate of the covariance matrix  $\Sigma_d$ . Such estimate is obtained every time the trigger on the hand-held probe is pulled. It must be highlighted that the uncertainty contribution obtained in this way is a function of the position (measured point) in the working volume. Furthermore, it only considers the repeatability and reproducibility contributions to the whole budget of uncertainty, but it does not take into account the accuracy contribution.

The two approaches are alternatives to each other. The first can be used for *a priori* optimisation of the measurement task and considers accuracy contributions, while the second has the advantage of considering the correlation terms but does not take into account accuracy. Both of them rely on the hypothesis of absence of systematic errors in the distance measurement. For the analysis in this article, the first approach is implemented.

#### 4.2. Uncertainty of calibration parameters

In all distributed measurement systems, calibration procedures are used to determine internal and external parameters of the system. This also applies to the MScMS for which the parameters are

- External parameters: That is, all the parameters that do not depend on the technology of measurement devices. External parameters of the MScMS are the location of the devices.
- Internal parameters: That is, the parameters that depend on the measurement devices technology. Internal parameters of the MScMS are the parameters defining bias corrections to Cricket measurements (Maisano and Mastrogiacomo 2010).

Being the result of the calibration, these parameters will be considered together into a single vector, that is,  $\xi$ .

Considerations similar to those reported for the measured distances apply to these parameters: their estimate is a function of several factors that depend on the procedure adopted for the calculation. There are several approaches to calibration that may be substantially different: some make use of different measuring instruments, others use calibrated reference artefacts or even a combination of the two (Franceschini et al. 2009a; Mastrogiacomo and Maisano 2010).

For this reason, it was decided to limit the development of the chain of uncertainty composition at this level. Being the result of the composition of several factors, the components of  $\xi$  were assumed to be normally distributed with mean equal to the true value and standard deviation estimated as follows.

As for  $\mathbf{d}$ , the uncertainty of  $\xi$  (in terms of standard deviation) can be obtained both through *a priori* and *posteriori* considerations.

Knowing the calibration procedure, it is possible to obtain a conservative estimate of the *a priori* variance associated with each of the parameters of the calibration. Then neglecting the correlation between the components of vector, the covariance matrix can be assumed to be diagonal with the elements on the diagonal equal to those estimates.

As an alternative, an estimation of  $\Sigma_\xi$  can be also obtained from the calibration procedure. In fact, this procedure is generally the optimisation of a specific error function. Assuming the absence of systematic errors, the estimation of  $\Sigma_\xi$  is given by the covariance matrix of the calibration results.

### 4.3. Uncertainty of 3D point coordinates

In order to obtain the uncertainty of 3D point coordinates, MLPU has to be applied to the general multilateration system:

$$\begin{bmatrix} \sqrt{(x_1 - x_p)^2 + (y_1 - y_p)^2 + (z_1 - z_p)^2} - C_1 \\ \sqrt{(x_2 - x_p)^2 + (y_2 - y_p)^2 + (z_2 - z_p)^2} - C_2 \\ \vdots \\ \sqrt{(x_n - x_p)^2 + (y_n - y_p)^2 + (z_n - z_p)^2} - C_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} \quad (5)$$

where

- $(x_p, y_p, z_p)$  are the unknown coordinates of point  $\mathbf{x}_p \equiv (x_p, y_p, z_p)$  to be localised, corresponding either to Cricket A or Cricket B on the hand-held probe
- $n \geq 4$  is the number of constellation Crickets in connection with the one positioned in  $\mathbf{x}_p \equiv (x_p, y_p, z_p)$
- $\mathbf{x}_1 = (x_1, y_1, z_1), \mathbf{x}_2 = (x_2, y_2, z_2), \dots, \mathbf{x}_n = (x_n, y_n, z_n)$  are the coordinates of the constellation Crickets, known by system calibration
- $d_1, d_2, \dots, d_n$  are the measured distances from  $x_p \equiv (x_p, y_p, z_p)$  to the constellation devices
- $C_1, C_2, \dots, C_n$  are the corrections of distance measurements obtained by performance-enhancing procedures (Maisano and Mastrogiacomo 2010).

In general, the correction term  $C_i$  can be a function of many variables such as the position of the point to be localised ( $\mathbf{x}_p$ ), the position of the constellation devices  $\mathbf{x}_i = (x_i, y_i, z_i)$  or other possible factors  $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots, \theta_m)$ :

$$C_i = f(\mathbf{x}_p, \mathbf{x}_i, \boldsymbol{\theta}) \quad (6)$$

Under this assumption, the left term of Equation (5) can be seen as a function of just  $\mathbf{x}_p, \mathbf{x}_i$  and  $\boldsymbol{\theta}$ . Thus, it can be rewritten as

$$F(\mathbf{x}_p, \xi) = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} \quad (7)$$

where  $\xi = [\mathbf{x}_1, \dots, \mathbf{x}_n, \boldsymbol{\theta}]^T$ . Equation (7) can then be linearised with a first-order Taylor expansion:

$$J_\xi \Delta \xi + J_p \Delta \hat{\mathbf{x}}_p = \Delta \mathbf{d} + \mathbf{v} \quad (8)$$

where

- $J_\xi \in \mathbb{R}^{n, 3n+m}$  is the Jacobian of  $F(\mathbf{x}_p, \xi)$  with respect to  $\xi$ , that is, each of the three coordinates of the  $n$  network devices and the  $m$  components of  $\boldsymbol{\theta}$

- $J_p \in \mathbb{R}^{n, 3}$  is the matrix of partial derivatives (Jacobian) of the functions in the left side of Equation (7) with respect to each of the 3D point coordinates (calculated in  $\mathbf{x}_p^0 = [x_p^0, y_p^0, z_p^0]^T$ )
- $\Delta \xi$  is the vector of the variations of the 3D coordinates of the  $n$  network devices and the possible factors affecting the correction functions.
- $\mathbf{v}$  is the vector of the residuals of distances between the point to be localised and the network devices
- $\Delta \mathbf{d}$  is the vector of the reduced observations, that is, the difference between measured and calculated distances:
$$\Delta d_i = d_i - \sqrt{(\hat{x}_i - x_p^0)^2 + (\hat{y}_i - y_p^0)^2 + (\hat{z}_i - z_p^0)^2}$$
- $\Delta \hat{\mathbf{x}}_p = \hat{\mathbf{x}}_p - \mathbf{x}_p^0$

Left-multiplying each member of Equation (8) for the same quantity  $J_p^T \mathbf{P}_d$  and neglecting the residual of the distances, the resulting normal equation becomes

$$N\Delta\hat{\mathbf{x}}_P = \mathbf{T} - \mathbf{J}_P^T \mathbf{P}_d \mathbf{J}_\xi \Delta\hat{\boldsymbol{\xi}} \quad (9)$$

where

$$\mathbf{N} = \mathbf{J}_P^T \mathbf{P}_d \mathbf{J}_P \quad (10)$$

$$\mathbf{T} = \mathbf{J}_P^T \mathbf{P}_d \Delta \mathbf{d} \quad (11)$$

$\mathbf{P}_d \in \mathbb{R}^{n,n}$  is the weight matrix, which can be defined as the inverse of the covariance matrix  $\boldsymbol{\Sigma}_d \in \mathbb{R}^{n,n}$  associated with the measured distances:

$$\mathbf{P}_d = \boldsymbol{\Sigma}_d^{-1} \quad (12)$$

The least-squares solution of Equation (9) is

$$\Delta\hat{\mathbf{x}}_P = \mathbf{N}^{-1} (\mathbf{T} - \mathbf{J}_P^T \mathbf{P}_d \mathbf{J}_\xi \Delta\hat{\boldsymbol{\xi}}) \quad (13)$$

So far, the combined point estimated covariance  $\hat{\boldsymbol{\Sigma}}_P$  estimated by applying MLPU to Equation (13) is

$$\hat{\boldsymbol{\Sigma}}_P = \mathbf{N}^{-1} + \mathbf{N}^{-1} (\mathbf{J}_P^T \mathbf{P}_d \mathbf{J}_\xi) \hat{\boldsymbol{\Sigma}}_\xi (\mathbf{J}_P^T \mathbf{P}_d \mathbf{J}_\xi)^T \mathbf{N}^{-1} \quad (14)$$

where  $\hat{\boldsymbol{\Sigma}}_\xi$  is the covariance matrix associated to the variable  $\boldsymbol{\xi}$ . From Equation (14), it is possible to understand how the overall uncertainty related to point  $P$  is strictly related to the position of the constellation devices through  $\hat{\boldsymbol{\Sigma}}_\xi$ . In other terms, defining a proper positioning for the constellation devices, it is possible to minimise the measurement uncertainty (Galetto and Pralio 2010).

Under the hypothesis of independence between the position of the reference network devices and  $\boldsymbol{\theta}$ ,  $\hat{\boldsymbol{\Sigma}}_\xi$  can be written as

$$\hat{\boldsymbol{\Sigma}}_\xi = \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_\varphi & \mathbf{0} \\ \mathbf{0} & \hat{\boldsymbol{\Sigma}}_\theta \end{bmatrix} \quad (15)$$

where  $\hat{\boldsymbol{\Sigma}}_\varphi$  and  $\hat{\boldsymbol{\Sigma}}_\theta$  are respectively the covariance matrices associated to the positions of the reference network devices and  $\boldsymbol{\theta}$ . In general, an estimate of matrix  $\boldsymbol{\Sigma}_\varphi$  can be obtained by the calibration procedure (Mastrogiacomo and Maisano 2010). Generally, if the position of each network device has been autonomously calibrated (i.e., separately from that of other network devices),  $\boldsymbol{\Sigma}_\varphi$  can be assumed to be a  $3 \times 3$  block diagonal matrix. If instead global calibration algorithms have been implemented, this assumption cannot be considered as true since the positions of different network devices may result correlated to each other (Mastrogiacomo and Maisano 2010). Similarly,  $\boldsymbol{\Sigma}_\theta$  is a  $3 \times 3$  block diagonal matrix which can be easily

defined during the calibration procedure (Maisano and Mastrogiacomo 2010).

#### 4.4. Uncertainty of probe tip coordinates

Knowing the positions of the two Crickets embedded in the mobile probe, the coordinate of the probe tip  $\mathbf{x}_V = (x_v, y_v, z_v)$  can be found as

$$\mathbf{x}_V = \mathbf{x}_A + \frac{\mathbf{x}_B - \mathbf{x}_A}{d(A-B)} d(A-V) \quad (16)$$

where  $\mathbf{x}_A$  and  $\mathbf{x}_B$  are respectively the positions of the two devices embedded in the probe,  $d(A-B)$  is their mutual distance and  $d(A-V)$  is the distance between device A and the probe tip. Assuming that the measurements of the coordinates of the two Crickets embedded on the probe and the two geometrical parameters of the probe ( $d(A-V)$  and  $d(A-B)$ ) are independent, MLPU can be applied to Equation (16) in order to obtain the covariance of the probe tip coordinates  $\boldsymbol{\Sigma}_V \in \mathbb{R}^{3,3}$  as

$$\hat{\boldsymbol{\Sigma}}_V = \mathbf{J}_V \boldsymbol{\Sigma}_\omega \mathbf{J}_V^T \quad (17)$$

where  $\mathbf{J}_V \in \mathbb{R}^{3,8}$  is the Jacobian of Equation (16), with respect to the coordinates of the two probe Crickets, and the parameters  $d(A-V)$  and  $d(A-B)$ .

$\boldsymbol{\Sigma}_\omega \in \mathbb{R}^{8,8}$  is the covariance matrix of parameters in Equation (16): assuming  $\mathbf{x}_A$ ,  $\mathbf{x}_B$ ,  $d(A-B)$  and  $d(A-V)$  to be independent from each other, it can be estimated as

$$\hat{\boldsymbol{\Sigma}}_\omega = \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_A & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\boldsymbol{\Sigma}}_B & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \hat{\sigma}_{d(A-V)}^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\sigma}_{d(A-B)}^2 \end{bmatrix} \quad (18)$$

where  $\hat{\boldsymbol{\Sigma}}_A$ ,  $\hat{\boldsymbol{\Sigma}}_B$  [obtained with Equation (16)] are respectively the estimated covariance matrices relating to point A and point B and  $\hat{\sigma}_{d(A-V)}^2$  and  $\hat{\sigma}_{d(A-B)}^2$  corresponding to  $d(A-V)$  and  $d(A-B)$  respectively (estimated during the probe calibration).

The diagonal elements of  $\hat{\boldsymbol{\Sigma}}_V$  provide an estimation of variances of the coordinates of the measured point. An evaluation of the expanded uncertainty is obtained by multiplying the corresponding standard deviations for a proper coverage factor  $k$  (usually equal to 2) (JCGM100:2008 2008):

$$\begin{aligned} U_{V,x} &= k \cdot \sqrt{\hat{\boldsymbol{\Sigma}}_{V,1,1}} \\ U_{V,y} &= k \cdot \sqrt{\hat{\boldsymbol{\Sigma}}_{V,2,2}} \\ U_{V,z} &= k \sqrt{\hat{\boldsymbol{\Sigma}}_{V,3,3}} \end{aligned} \quad (19)$$

The three parameters can be combined into the corresponding 3D radial uncertainty:

$$U_V = k \cdot \sqrt{\hat{\Sigma}_{V,1,1} + \hat{\Sigma}_{V,2,2} + \hat{\Sigma}_{V,3,3}} \quad (20)$$

### 5. Test of the method

The model which is proposed in general terms in the previous sections has been tested on MScMS. In its current configuration, the MScMS network of wireless devices is composed of seven devices uniformly distributed on the ceiling of a laboratory, thus defining a working volume of about  $3 \times 3 \times 2$  m (see Figure 4). Their position has been calibrated using a laser tracker (FARO) (Mastrogiacomo and Maisano 2010).

The hand-held probe is characterised by  $d(A-V) = 540.00$  mm and  $d(A-B) = 450.00$  mm (see Figure 4). These distances have been evaluated by means of a CMM (DEA IOTA 0101), so that the corresponding estimates of standard uncertainties are respectively  $\hat{\sigma}_{d(A-V)} = 0.10$  mm and  $\hat{\sigma}_{d(A-B)} = 0.10$  mm.

The following functions ( $C_i$ ) are used to correct the distances (Maisano and Mastrogiacomo 2010; Franceschini et al. 2011):

$$C_i(x_i, x_P) = \alpha_0 + \alpha_1 d_{iP} + \alpha_2 \gamma_{iP} - \alpha_3 d_{iP} \gamma_{iP} \quad (21)$$

where  $\alpha_j$ , with  $j = 1, 2, 3$  are the coefficients of the model proposed by Maisano and Mastrogiacomo and

$$d_{iP} = \sqrt{(x_i - x_P)^2 + (y_i - y_P)^2 + (z_i - z_P)^2} \quad (22)$$

$$\gamma_{iP} = \cos^{-1} \left( \frac{z_i - z_P}{\sqrt{(x_i - x_P)^2 + (y_i - y_P)^2 + (z_i - z_P)^2}} \right) \quad (23)$$

Notice how  $C_i$  depends on the point to be localised ( $x_P$ ), the position of the constellation devices  $x_i$  and also on the coefficients of the correction model ( $\alpha_i$ ). Thus, referring to Section 4,  $n = 7$  and  $m = 4$ .

To provide a preliminary validation of the model, the following test has been made. A set of 50 points randomly selected within the working volume have been measured using MScMS for a total of 20 repetitions for each point. The variability of results was compared with the covariance matrix obtained from the model. Figure 5 shows the comparison. For ease of synthesis, the graph shows the radial uncertainty [see Equation (20) with  $k = 1$ ].

In order to test the hypothesis of equality between the variances equivalent to the empirical and the model 3D radial uncertainty (respectively  $U_{V, \text{empirical}}^2$  and  $U_{V, \text{model}}^2$ ), a Fisher test with 5% confidence level has been used. Considering the directions of GUM (JCGM100:2008

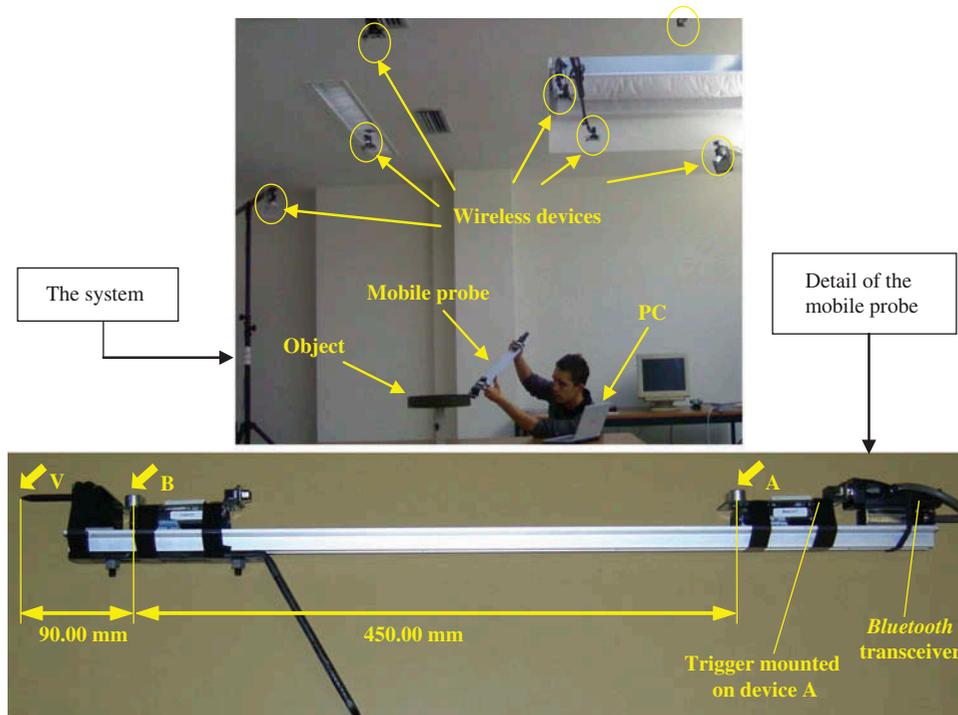


Figure 4. The MScMS prototype and its hand-held probe. Its physical dimensions have been evaluated with CMM (DEA IOTA 0101).

2008),  $v_1 = 30$  (assumed) and  $v_2 = 18$  degrees of freedom were used respectively for the model and the empirical data. Under these assumptions, the threshold values for the test resulted to be  $F_{crit,U}(\alpha = 0.975, v_1 = 30, v_2 = 18) = 2.44$  and  $F_{crit,L}(\alpha = 0.025, v_1 = 30, v_2 = 18) = 0.45$ . For 49 out of 50 points, the test resulted successful, that is, the ratios between the variances were within the threshold values:

$$F_{crit,L}(v_1, v_2) \leq \frac{U_{V,model}^2}{U_{V,empirical}^2} \leq F_{crit,U}(v_1, v_2) \quad (24)$$

The only case out of 50 (point 35, see Figure 5) for which the test rejected the null hypothesis of equality of variances is probably a false positive, that is, a case where the test rejects a true null hypothesis. In all other cases, the test can be seen as there is no evidence to exclude the null hypothesis.

Another important aspect arising from the test is the fact that the expected values of the overall uncertainty are in the order of 1 cm. This relatively high value is mainly due to the uncertainty related to the distance measurements. On the other hand, the contributions deriving from the uncertainty of probe geometrical parameters are lower compared to the others.

For many applications in LSDM, this value of overall uncertainty is hardly acceptable, so that a substantial reduction is required, for example, by enhancing the performance of the network devices or replacing the US sensors with a more accurate technology.

## 6. An application case

As noted in the previous section, probably the major defect of MScMS is its accuracy. However, despite its

current level of performance, this and similar localisation systems based on ultrasound sensors have been used in many contexts where the accuracy requirements are compatible with those offered by the technology:

- Indoor navigation/localisation. Many are the examples of ultrasound systems applied to guide users within closed environments such as offices, factories or museums. These systems are generally able to target users to different types of resources such as paintings, exhibition halls, as well as fire extinguishers, emergency exits or even first aid boxes (Balakrishnan et al. 2003; Teller, Chen, Balakrishnan 2003; Galetto et al. 2010).
- Warehouse management. For this type of applications, ultrasound systems are used with different purposes, such as the dimensional control of the goods and positions in stock or for operators guidance towards specific goods contained in the warehouse (Franceschini et al. 2009b).
- Robot control. Some attempt to use such systems for robot control have been done, also in the direction of cooperative coordination of fleets of robots (Moore, Leonard, and Rus 2004).
- Generic applications of indoor/outdoor positioning. In addition to the above cited applications, ultrasonic tracking systems have been used for the localisation of objects/operators in various other contexts (Jiménez and Seco 2005).

MScMS has been successfully tested in some of the application fields proposed earlier. This section proposes an application example of MScMS for the automation of the warehouse management of a shipping company. For this purpose, one of the receiving docks of the company

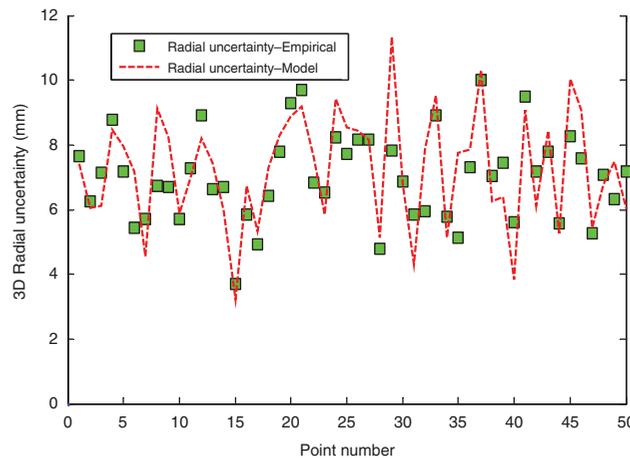


Figure 5. Empirical versus model radial uncertainty ( $k = 1$ ).

was used as a test bench. An area of about  $5 \times 5 \times 2$  m was equipped with a set of nine network devices whose position was calibrated with the aid of a laser tracker.

The system was installed to be used for the dimensional characterisation of the received goods. Potentially the system could be easily integrated into the inventory management software, thus allowing automatic location assignment for each received package based on space availability and inventory requirements.

In this case, the specific requirement was that of a cheap system able to measure the dimensions of complex shapes up to 5 m with an accuracy of about 10 mm. To verify whether the system was able to meet the accuracy requirement, the model proposed in this article was applied with the following assumptions. Neglecting the correlations, the covariance matrix of measured distances was estimated through Equation (4) as

$$\hat{\Sigma}_d = \sigma_0^2 \mathbf{I} = 25 \cdot \mathbf{I}_9 \text{ mm}^2 \quad (25)$$

That is, a diagonal matrix with  $\sigma_0 = 5$  mm and  $\mathbf{I}_9$  the identity matrix of size 9 (the number of network devices). The covariance matrix of calibration parameters was estimated through Equation (15) as

$$\hat{\Sigma}_\xi = \begin{bmatrix} \hat{\Sigma}_\varphi & \mathbf{0} \\ \mathbf{0} & \hat{\Sigma}_\theta \end{bmatrix} \quad (26)$$

where  $\hat{\Sigma}_\varphi$ , that is, the covariance matrix associated to the positions of the reference network devices, was modelled as

$$\hat{\Sigma}_\varphi = 10^{-8} \cdot \mathbf{I}_{27} \text{ mm}^2 \quad (27)$$

On the other hand, an estimation of  $\hat{\Sigma}_\theta$ , that is, the covariance matrix of  $\theta$ , is given in Maisano and Mastrogiacomo (2010):

$$\hat{\Sigma}_\theta = \begin{bmatrix} (6 \cdot 10^{-1})^2 & 0 & 0 & 0 \\ 0 & (2 \cdot 10^{-4})^2 & 0 & 0 \\ 0 & 0 & (4 \cdot 10^{-4})^2 & 0 \\ 0 & 0 & 0 & (1 \cdot 10^{-5})^2 \end{bmatrix} \quad (28)$$

Since probe geometric parameters were assessed by means of CMM (DEA – IOTA 0101), the related uncertainty was set to

$$\sigma_{d(A-B)}^2 = \sigma_{d(A-V)}^2 = \sigma_{d(B-V)}^2 = 10^{-4} \text{ mm}^2 \quad (29)$$

A layout with nine network devices uniformly distributed in an area of  $5 \times 5 \times 2$  m was tested on 150 points randomly taken within by using the uncertainty model

proposed here. The worst-case result ( $U_v = 9$  mm) suggested that the layout was enough to satisfy the accuracy requirements.

As it could be expected, the use of ultrasonic sensors results in a rather poor value of uncertainty. In such conditions, the proposed methodology allows to *a priori* evaluate whether the measurement method satisfies the required uncertainty constraints or if the use of other more accurate (but maybe more expensive) methods is necessary. In the case exemplified, an uncertainty equal to  $U_v = 9$  mm is acceptable, but would certainly not be allowed for more refined measurements, such as those of precision mechanical parts.

## 7. Conclusions

This article presents a method based on MLPU for the evaluation of the uncertainty related to the coordinates of each point measured by MScMS. The method provides a covariance matrix that uniquely defines the variability of the measurement. It was tested on a set of points providing positive results. Even if it has been developed specifically for MScMS, the method can be easily extended to similar systems based on multilateration.

From the point of view of the application, the method provides some advantages. Offering an estimate of measurement uncertainty, it can be used to weigh the points when the purpose of the measurement is to reconstruct complex geometries. Dually, it can be useful for defining the location of the network devices in order to optimise the results of the measurement. Finally, it can be helpful for the definition of an appropriate measurand or probe position.

Currently the major limitations of the proposed method lie in the fact that the development of the uncertainty composition chain is arrested at measured distances and calibration parameters. These variables are surely affected by several other factors, such as thermal or humidity gradients, velocity of air flow, composition of the air and so on. Although difficult to control, if modelled, these parameters could lead to a more accurate estimate of uncertainty.

Future developments include the extension of the methodology to similar systems based on triangulation rather than multilateration. These systems generally represent a step forward in terms of metrology performance.

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